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Abstract

To understand the potentials of bayesian panel data analysis, simulated data are used to estimate a random effects model. Prior Gaussian distributions of various precision are used to understand the influence of the prior information on continuous, discrete, time varying and time constant variables. It is demonstrated that parameters of dummy variables are far more sensitive to priors than parameters of continuous variables. Time varying variables are less sensitive than their time constant counterparts. It is concluded that bayesian panel data analysis is of interest if data do not provide enough information and if adequate extraneous information is available.

1 Introduction

The idea of this paper is to build a theoretical framework that combines information provided by data and information from extraneous sources by applying bayesian statistics. In Bayesian frameworks the prior information about the parameters is summarized in terms of a probability density function and once the data becomes available, this prior is updated. The inference is then made from the posterior distribution, which is obtained

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by combining prior and likelihood derived from the data. The data used for this simulation have panel structure while the the prior information is embodied in a distribution. The latter can be interpreted as information provided by experts, results from previous research or results from another scientific discipline. The prior information is then updated by the data. The study focuses in particular on the influence of prior information on the estimated effects of different factors. It is done for panel data since it is an often appearing structure of data.

The next chapter gives an overview about panel data analysis to be followed by a chapter that focuses on Bayesian panel data analysis. The bayesian panel data model is explained there. In chapter 4 the simulated variables are presented. This is followed by the discussion of the results and finally by the conclusion. The appendices include the code of the WinBUGS Model, the tables with the results of the estimation and some illustration. The models were estimated with the freely available statistical software packages R [8] and WinBUGS [10].

2 On panel data models

Analysis of panel data requires to take account of the panel specific structure of several observations for each individual. If ordinary least-square (OLS) regression is used, the standard assumptions must be fulfilled (see for example Greene [4], page 10). But it is unlikely, that the error terms are uncorrelated between individuals and over time. The two most popular approaches to take account of the special time structure are fixed and random effects models. The fixed effects model has individual-specific dummy variables and it is assumed that the differences across units can be captured in differences in this constant term. The model can be reformulated by taking the deviation of the mean of all explaining variables instead of including individual specific dummy variables by applying the Frisch-Waugh Theorem (see Frisch and Waugh [2]). This reformulation has no effect on the results of the estimated parameters but since the number of variables is reduced this formulation has computational advantages. The most appealing aspect of the fixed effect model is that it is robust to the omission of any relevant time-invariant regressors. On the other hand, time-invariant regressors cannot be estimated because their influence is captured in the individual specific dummy or, in the case of the simplified formulation, because the variables are zero. The second most popular approach is the random effects model. It is assumed that the individual specific effects are uncorrelated with the explaining variables. Data therefore do not carry useful information about the error term. A variance-covariance matrix can be used to describe how much certain observation depend on each other. In a frequentist framework, this is identical to a Generalized Leased Square (GLS) estimation where the variance-covariance matrix for the feasible GLS can be taken either from a fixed effects or OLS regression. For a detailed discussion see e.g. Baltagi [1]. Lancaster [7] gives on page 270 a nice example of what the random effects model assumption imply:

> A classic example is that of an agricultural production function in which the output of the farm, bushels of wheat for example, is taken to depend on the levels of factor inputs such as the amounts of labour and capital that are used, so that, for example
>
> $$q = \alpha + \beta_1 k + \beta_2 l + \epsilon$$
>
> where q is output, k, l are measures of capital and labour inputs (usually measured in logarithms) and ϵ summarizes all other sources of variations in output. In a randomized experiment k and l would be allocated to a farm by random number generator and so they can be plausibly assumed independent of the unmeasured determinants of output represented by ϵ. In a controlled experiment matters will be arranged so that when k and l are changed ϵ remains the same. But without these conditions we surely must believe that farmers choose their capital and labour inputs and that they do so in the light of some of the factors that enter into ϵ. These may include particular features of the land being farmed, of the weather, of the anticipated behavior of competitors, etc. This line of thought suggests that independence of ϵ and k, l is unpersuasive.

If the assumption of independence between the explaining variables and the error terms is not met, the model may suffer from inconsistency due to this correlation (Hausman and Taylor [6]). Unfortunately, the independence can not be tested directly. But Hausman [5] developed a test based on the idea that under the hypothesis of no correlation, both – fixed and random effects model – are consistent. But under this hypothesis the fixed effects model is inefficient, whereas under the alternative hypothesis, the fixed effects model is consistent, but the random effects model is not. Therefore, under the null hypothesis with no correlation, the two estimates should not differ systematically, and a test can be based on the difference. A justification of the random effects assumptions can be based on the Hausman test. However, to estimate time constant variables even if the assumptions are not met, Hausman and Taylor [6] introduced an instrumental variable random effects model. This allows to estimate a random effects model even if the unit specific constant terms are correlated with the explaining variables by using instrumental variables. In their model they distinguish between variables that are uncorrelated with the error term and those that are correlated. The tricky part is the determination which of the variables are correlated with the error term. Since the determination has a major impact on the estimated parameters, and there are no tests available, its application is problematic.

3 Bayesian panel data analysis

The models described above have been mainly applied in a frequentist framework. But it is also possible to build models with the same theoretical characteristics using Bayesian statistics. According to the convention in Bayesian statistics not the variance but the precision, which is the inverse of the variance, is used through out the text. The normal distribution is therefore given as $N \sim (mean, precision)$ in stead of $N \sim (mean, variance)$.

3.1 Fixed effects model

The fixed effects model can be understood as an OLS model with the matrix of explaining variables, X, being augmented by vectors of dummy variables for each individual. The fixed effects model can then be written as

$$y_i = X_i\beta + \alpha_i j_T + \epsilon_i, \qquad \epsilon_i \perp X_i, \beta, \alpha_i, \epsilon_i \sim n(0, \tau I_T). \tag{1}$$

The dependent variable y_i is a vector of length T giving the consecutive y values for agent i , X_i is a $T \times k$ matrix and β is a $k \times 1$ vector of coefficients common to all agents. The error term ϵ_i is of length $T \times 1$ and is normal, homoscedastic, not autocorrelated and independent of X_i, α_i, and β. The term I_T represents the identity matrix of size T. Equation 1 can be written for all agents as

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = \begin{bmatrix} X_1 & j_T & 0 & \cdot & \cdot & \cdot & 0 \\ X_2 & 0 & j_T & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & j_T & 0 \\ X_N & 0 & \cdot & \cdot & \cdot & 0 & j_T \end{bmatrix} \begin{bmatrix} \beta \\ \alpha_1 \\ \cdot \\ \cdot \\ \alpha_N \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \cdot \\ \cdot \\ \epsilon_N \end{bmatrix}, \tag{2}$$

where y is of size $N \times T$, the matrix with the explaining variables is of order $NT \times (k + N)$ and the coefficient vector is of length $k + N$.

For the resulting model a likelihood function and a priori distribution must be determined. Combining the matrix of the independent variables, X, and the matrix of the dummy variables, $\alpha_i j$, in a new variable Z and defining the respective parameter vector as δ, the fixed effects model can be written as

$$y = Z\delta + \epsilon, \;\; \epsilon | Z, \epsilon \sim n(0, \tau I_{NT}). \tag{3}$$

The likelihood function of this model is then given by (see eg. Lancaster [7]),

$$\ell(\delta, \tau) \propto \tau^{NT/2} exp \left\{ -(\tau/2)(y - Z\delta)'(y - Z\delta) \right\}. \tag{4}$$

An non-informative prior, which is a distribution with a very high variance, does not impose strong preconditions on the parameter and as a result the posterior is almost completely determined by the data. It makes sense if nothing is known of the parameter before the experiment. If, however, such information is available it can be incorporated into an informative prior. The posterior distribution is proportional to the product of prior and likelihood. Once it is evaluated, inferences may be drawn, using e.g. measures of central tendency (mean, median) and variation.

3.2 Random effects model

The fixed effects model assumes that the individual effects, α_i, are distributed uniformly. This assumption is made implicitly by not estimating a common mean for the α_i. But in many cases it is more reasonable to assume that the constant are all similar and are located around a common value. This can be done in a hierarchical Bayesian random effects model. The individual effects are assumed to vary normally around $\bar{\alpha}$ with precision ϕ. The two resulting equations are:

$$y_i = X_i\beta + \alpha_i j_T + \epsilon_i, \tag{5}$$

$$\alpha_i = \bar{\alpha} + \eta_i \tag{6}$$

where ϵ_i and η_i are independently normally distributed with precision τ and ϕ respectively. By taking equation 5 and 6 together

$$y_i = X_i\beta + \alpha j_T + (\epsilon_i + \eta_i j) \tag{7}$$

is derived. Equation 7 can be expressed less compressed as

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = \begin{bmatrix} X_1 & j_T \\ X_2 & j_T \\ \cdot & \cdot \\ \cdot & \cdot \\ X_N & j_T \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \end{bmatrix} + \begin{bmatrix} \epsilon_1 + j_T\eta_1 \\ \epsilon_2 + j_T\eta_2 \\ \cdot \\ \cdot \\ \epsilon_N + j_T\eta_N \end{bmatrix}. \tag{8}$$

The two parts of the error term are normally distributed around zero and the precision and the variance-covariance matrix can be determined (see Lancaster [7], page 192).

The likelihood function for this model is then given by

$$\ell(\beta, \tau, \alpha_i) \propto \tau^{NT/2} exp\left\{ -(\tau/2) \sum_{i=1}^{N} (y_i - X_i\beta - \alpha_i j)'(y_i - X_i\beta - \alpha_i j) \right\} \tag{9}$$

where α_i is as in equation 6.

Within Bayesian frameworks all the parameters should be assigned a prior distribution. The posteriori distribution is proportional to the product of priori distribution and the likelihood function.

4 Simulation

The goal of the simulation is to check how useful a Bayesian analysis is for panel data with prior information. Of special interest is the behavior of variables that do not change over time since their variation is limited and prior information could help to improve the estimation. All variables that are time constant for an individual cannot be estimated in a fixed effects model. For the fixed effects model, the result is only determined by the prior. Hence, if the prior for a time constant variable is uninformative, the starting value determines the results. The random effects model is different since the individual dummies do not capture all the time constant influences but are forced to vary around a mean. This leaves some room for individual constant terms to be estimated without running into an identification problem. But the variation of the time constant variables is smaller than the variation of a freely varying variable since the number of different observations is substantially smaller.

The data simulated consist of 400 individuals. For each individual 5 observations are simulated. Hence the data consists of 2000 observations in total. There are 8 explaining variables, x_1 to x_8. The variables are described in table 1. The variables x_1 to x_4 are all normally distributed around zero with standard deviation 100. The variables x_5 to x_8 are binary random variables which are equal to 1 with probability 0.5 and are equal to 0 otherwise. Variables x_1 and x_5 are constant over time for each individual. Variables x_2 and x_6 are time invariant for 99.5% of the individual. The variables x_3 and x_7 are time invariant for 95% of the individuals. And finally the variables x_4 and x_8 vary freely in time. All other variables and parameters of the model necessary to calculate the dependent variable y were simulated as well: the coefficients of the variables, $beta1$ to $beta8$ were sampeld from a normal distribution with mean 5 and standard deviation of

variable	type	over time	parameter
x_1	continuous	constant	$N \sim (0, 100)$
x_2	continuous	99.5% constant	$N \sim (0, 100)$
x_3	continuous	95% constant	$N \sim (0, 100)$
x_4	continuous	varying	$N \sim (0, 100)$
x_5	discrete	constant	$B \sim (400 * 5, 0.5)$
x_6	discrete	99.5% constant	$B \sim (400 * 5, 0.5)$
x_7	discrete	95% constant	$B \sim (400 * 5, 0.5)$
x_8	discrete	varying	$B \sim (2000, 0.5)$

Table 1: Description of variabels

0.005. The error term, ϵ, is sampled from a normal distibution with mean zero and standard deviation 10. The mean of the constants, $\bar{\alpha}$, is determined to be 50 and the variance of its normal distribution, ϕ, as $\sqrt{10}$. Having determined these variables, the dependent variable, y, was calculated.

To observe the behavior of the posterior distribution under differently precise prior distributions eight models are estimated. All the priori distributions of the β coefficients are identical within a model. This allows to compare the influence of the data. In order to compare the effect of differently precise prior information, the precision of the β parameters' priors are different between the models. The variance in the different models is between 10000 and 1. The mean of the priori distribution of the parameters is in all models 50. The mean of all (simulated) beta parameters from the data is around 5. An overview of the prior information of the parameters in different models is provided in table 2. The model written in WinBUGS code can be found in appendix A. There is also documented that phi (ϕ) has mean 1 and variance 0.1. This relatively precise distribution is necessary as the model will oterhwise converge only very slowly (see Lancaster [7], page 293).

distribution	precision
$N \sim (50, 10000)$	0.0001
$N \sim (50, 100)$	0.01
$N \sim (50, 50)$	0.02
$N \sim (50, 25)$	0.04
$N \sim (50, 10)$	0.1
$N \sim (50, 5)$	0.2
$N \sim (50, 2)$	0.5
$N \sim (50, 1)$	1

Table 2: Priori distributions for parameters of different models.

5 Results

The results of the estimations are summarized in tables 3 to 10 and in the figures in appendix C. As can be seen from the tables, the potential scale reduction factor, $\hat{R}$, of all parameters of interest is close to 1. The potential scale reduction factor, $\hat{R}$, helps to monitor the convergence of the iterative simulation by estimating the factor by which the scale of the current distribution of the parameter might be reduced if the simulations were continued in the limit $n \to \infty$. After all convergence has been reached, $\hat{R}$ is 1. Gelman at al. (page 332) [3] argue that approximate convergence of an parameter has been reached when $\hat{R}$ is smaller than 1.2. Convergence was only feasible through 50,000 Markov Chain Monte Carlo (MCMC) iterations. This is in particular true for the models with precise prior as for them more iterations were required to reach convergence. This probably happened because the data and prior information are quite far apart if their precision is taken into account. Hence the resulting posteriori distribution was bimodal and this can cause problems for the convergence of MCMC simulations.

Comparing the parameters for the continuous variables and those for the dummy variables, as depicted in the figures in appendix C, it becomes obvious that the influence of the prior is much stronger in the case of dummy variables. This is due to the higher variance of the continuous variables. This higher variance is enough to dominate the prior. The coefficients of the continuous variables in the different models are all around 5, while the coefficient of the dummy variables are between 5 and 50, depending on the precision of the prior. Also the standard deviation of the parameters is by far higher for the dummy variable coefficients.

Interestingly, the parameters for *beta*1, *beta*3 and *beta*4 are slightly underestimated in all models. From the tables can be seen that for models with a very uninformative prior (precision between 0.0001 and 50), the standard deviations are practically identical. In models with more informative priors, the coefficients for variables with higher variance (*beta*4 and *beta*3) have a smaller standard deviation. Here the data are more dominant because of their higher variance. The figures show that the influence of the prior is very limited on the mean of the posteriori distribution[1]. But the variance increases with increasing precision of the prior. At the same time, more variation in the variable reduces this increase.

For the discrete dummy variables the conclusions are different. The figures in the tables show a substantial influence of the prior information. The more precise the prior, the bigger its influence. Therefore the means of the posterior distribution of models with higher precision are closer to 50. As for the continuous variables, this effect is stronger for variables with less variation. Also the standard deviation is higher for more precise priors.

As can be seen from table 3 to table 10, the parameter for *alphabar* ($\bar{\alpha}$) reduces with increasing precision of the priors of the β parameters. This is due to the fact that the estimated β parameters increase while y remains the same. Hence *alphabar* must reduce. The precision of *alphabar*, which is *phi*, is close to 1 for the models with non-informative priors and almost 0 for the models with precise priors for the *beta* parameters. The parameter *tau* (τ) which estimates the precision of the estimation, reduces slightly with increasing precision of the priors of the *beta* parameters.

For each model WinBUGS automatically evaluates Deviation Information Criteria (DIC; Spiegelhalter [9]) which may the be used for model comparison.

[1]It is even smaller for a higher precision in the case of *beta*3.

6 Conclusions

Models commonly used in frequentist panel data analysis can also be constructed in Bayesian statistics. The advantage of a Baysian approach is that prior information can systematically be included in the analysis. The influence of the prior depends mainly on its precision and on the variance of the variables. The higher the precision of the priori distribution, the bigger its influence. But also the structure of the data has an influence on the posterior distribution. If there is more variation within a variable, the influence of the data is bigger. These results have the following consequences for a random effects panel data analysis. First, discrete data modeled by dummy variables typically have a much smaller variance than continuous variables. Hence, informative priors have a much higher influence on dummy variable parameters than on continuous variable parameters. Secondly the influence of the prior is bigger for variables that are constant over time. The reason is that these data provide less information as there cannot be more different observations than there are individuals. For these variables it is especially interesting to find appropriate prior information since not that much information can be gained from the data. But it must also be taken extra care since it is important that the prior distribution is appropriate. The potential improvement of the estimation results through correct prior information can also be a disadvantage if the prior information is wrong.

This can be summarized in two potential advantages of the Bayesian model. Firstly, information from different sources can systematically be included in the model. Hence information gathered by other scientists can be used as starting point. Secondly the prior information can help out where the data have weaknesses. Data weakness can either occur from unreliable data sources or from data with insufficient variance. In other words, if we know better than the data, the Bayesian model allows to use this knowledge within an econometric model.

A possible application is agricultural economics. Here different scientific disciplines determine parameters of interest, such as the influence of a fertilizer on the growth of a plant. While plant breeders tend to gather their data from controlled experiments, economists use data from surveys. Bayesian statistics allows to merge information from these two sources systematically. This also applies to time constant variables. The soil of a farm's fields is typically constant over time. If there is a limited number of soil types the econometrician would use dummies to model differences in the crop yield due to the soil types. If there are no correlations with other data that disturb the dummy variable estimation, the results will be correct. But due to their simple structure time constant dummy variables are often correlated with other variables. These other variables might or might not be part of the specified model. This makes the dummy variables vulnerable for biases. If non-data information about the yields for different soil types exist, this can help to solve this identification problem. This is important because a biased parameter estimate can also have implications for other parameter estimations if there is correlation.

The model presented here would use the plant breeder's findings as priors and update them with panel survey data. But the set up could also be the other way round, such that the survey data provide the prior information and the breeders finding is used as an update.

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A WinBUGS Model Code

```
model{                     #def. of the model
for(n in 1:N){             #N total observations
y[n]~dnorm(mu[n],tau)
mu[n]<-
beta1*x1[n]+
beta2*x2[n]+
beta3*x3[n]+
beta4*x4[n]+
beta5*x5[n]+
beta6*x6[n]+
beta7*x7[n]+
beta8*x8[n]+
alpha[farm[n]]}
for (f in 1:T){            #T obs. per individual
alpha[f]~dnorm(alphabar,phi)}

alphabar~dnorm(0,.0001) #def. prior distributions
beta1~dnorm(50,pre)     #dnorm(mean,precision)
beta2~dnorm(50,pre)     #precision=1/variance
beta3~dnorm(50,pre)
beta4~dnorm(50,pre)
beta5~dnorm(50,pre)
beta6~dnorm(50,pre)
beta7~dnorm(50,pre)
beta8~dnorm(50,pre)
tau~dgamma(0.01,0.01)   #gamma(r,mu)
phi~dgamma(10, 10)      #mean=r/mu, var=r/mu^2}
```

B Tables of Results

Table 3: Results for β prior distributions $N \sim (50, 10000)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	48.152	0.753	46.760	49.770	1.015
beta1	4.988	0.003	4.982	4.994	1.000
beta2	5.000	0.003	4.994	5.006	1.000
beta3	4.998	0.003	4.992	5.004	1.003
beta4	4.998	0.003	4.992	5.004	1.004
beta5	5.646	0.666	4.365	6.977	1.003
beta6	6.762	0.639	5.501	7.965	1.002
beta7	6.475	0.627	5.253	7.734	1.000
beta8	4.687	0.641	3.353	5.909	1.001
tau	0.005	0.000	0.005	0.005	1.005
phi	0.947	0.306	0.434	1.606	1.000
deviance	16280.860	9.796	16260.000	16300.000	1.003

Table 4: Results for β prior distributions $N \sim (50, 100)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	47.780	0.752	46.390	49.400	1.015
beta1	4.988	0.003	4.982	4.994	1.000
beta2	5.000	0.003	4.994	5.006	1.000
beta3	4.998	0.003	4.992	5.004	1.003
beta4	4.998	0.003	4.992	5.004	1.004
beta5	5.842	0.665	4.568	7.172	1.003
beta6	6.943	0.637	5.686	8.143	1.002
beta7	6.649	0.626	5.427	7.905	1.000
beta8	4.864	0.640	3.532	6.084	1.001
tau	0.005	0.000	0.005	0.005	1.005
phi	0.946	0.306	0.432	1.605	1.000
deviance	16281.170	9.830	16260.000	16300.000	1.001

Table 5: Results for β prior distributions $N \sim (50, 50)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	47.407	0.751	46.020	49.030	1.015
beta1	4.988	0.003	4.982	4.994	1.000
beta2	5.000	0.003	4.994	5.006	1.000
beta3	4.998	0.003	4.992	5.004	1.003
beta4	4.998	0.003	4.992	5.004	1.004
beta5	6.038	0.664	4.772	7.364	1.003
beta6	7.125	0.636	5.872	8.323	1.002
beta7	6.824	0.625	5.602	8.076	1.000
beta8	5.042	0.639	3.714	6.261	1.001
tau	0.005	0.000	0.005	0.005	1.005
phi	0.945	0.306	0.432	1.603	1.000
deviance	16282.210	9.793	16260.000	16300.000	1.001

Table 6: Results for β prior distributions $N \sim (50, 25)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	46.667	0.750	45.290	48.280	1.015
beta1	4.988	0.003	4.982	4.994	1.000
beta2	5.000	0.003	4.994	5.006	1.000
beta3	4.998	0.003	4.992	5.004	1.003
beta4	4.998	0.003	4.992	5.004	1.004
beta5	6.426	0.662	5.162	7.743	1.003
beta6	7.484	0.635	6.238	8.680	1.002
beta7	7.171	0.623	5.950	8.418	1.000
beta8	5.395	0.637	4.070	6.611	1.001
tau	0.005	0.000	0.005	0.005	1.005
phi	0.941	0.306	0.430	1.601	1.000
deviance	16285.540	10.383	16260.000	16300.000	1.002

Table 7: Results for β prior distributions $N \sim (50, 10)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	44.480	0.754	43.090	46.101	1.015
beta1	4.988	0.003	4.982	4.994	1.000
beta2	5.000	0.003	4.994	5.006	1.000
beta3	4.998	0.003	4.991	5.003	1.002
beta4	4.998	0.003	4.992	5.004	1.004
beta5	7.572	0.661	6.300	8.904	1.003
beta6	8.548	0.633	7.310	9.745	1.002
beta7	8.197	0.622	6.975	9.445	1.000
beta8	6.440	0.636	5.118	7.662	1.001
tau	0.005	0.000	0.005	0.005	1.005
phi	0.916	0.306	0.410	1.583	1.000
deviance	16310.490	14.452	16280.000	16340.000	1.000

Table 8: Results for β prior distributions $N \sim (50, 5)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	40.791	0.783	39.350	42.460	1.014
beta1	4.988	0.003	4.981	4.994	1.000
beta2	5.000	0.003	4.994	5.006	1.000
beta3	4.997	0.003	4.991	5.003	1.003
beta4	4.998	0.003	4.992	5.005	1.004
beta5	9.500	0.672	8.211	10.870	1.003
beta6	10.342	0.642	9.084	11.550	1.002
beta7	9.933	0.630	8.712	11.200	1.000
beta8	8.203	0.643	6.868	9.438	1.001
tau	0.005	0.000	0.004	0.005	1.004
phi	0.826	0.313	0.292	1.509	1.000
deviance	16396.540	24.212	16350.000	16440.000	1.002

Table 9: Results for β prior distributions $N \sim (50, 2)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	−6.515	1.953	−10.492	−2.813	1.000
beta1	4.985	0.014	4.959	5.012	1.001
beta2	5.005	0.013	4.980	5.029	1.001
beta3	4.991	0.009	4.974	5.009	1.002
beta4	4.998	0.004	4.990	5.005	1.003
beta5	41.231	1.454	38.179	43.991	1.002
beta6	39.803	1.371	37.130	42.500	1.002
beta7	34.194	1.155	31.989	36.490	1.001
beta8	15.488	0.755	14.000	17.050	1.000
tau	0.004	0.000	0.004	0.004	1.003
phi	0.001	0.000	0.001	0.002	1.001
deviance	16706.210	48.502	16610.000	16800.000	1.000

Table 10: Results for β prior distributions $N \sim (50, 1)$

	mean	stand. dev.	2.5%	97.5%	$\hat{R}$
alphabar	−21.692	1.871	−25.402	−18.199	1.001
beta1	4.995	0.016	4.963	5.027	1.001
beta2	5.011	0.015	4.981	5.041	1.001
beta3	4.993	0.010	4.973	5.015	1.002
beta4	4.999	0.005	4.990	5.008	1.003
beta5	46.484	1.001	44.508	48.440	1.004
beta6	45.767	0.994	43.820	47.670	1.003
beta7	42.582	0.892	40.820	44.341	1.001
beta8	25.487	0.831	23.870	27.160	1.000
tau	0.003	0.000	0.003	0.003	1.003
phi	0.001	0.000	0.001	0.001	1.001
deviance	17343.140	61.536	17229.750	17460.000	1.000

C Comparison of Parameters

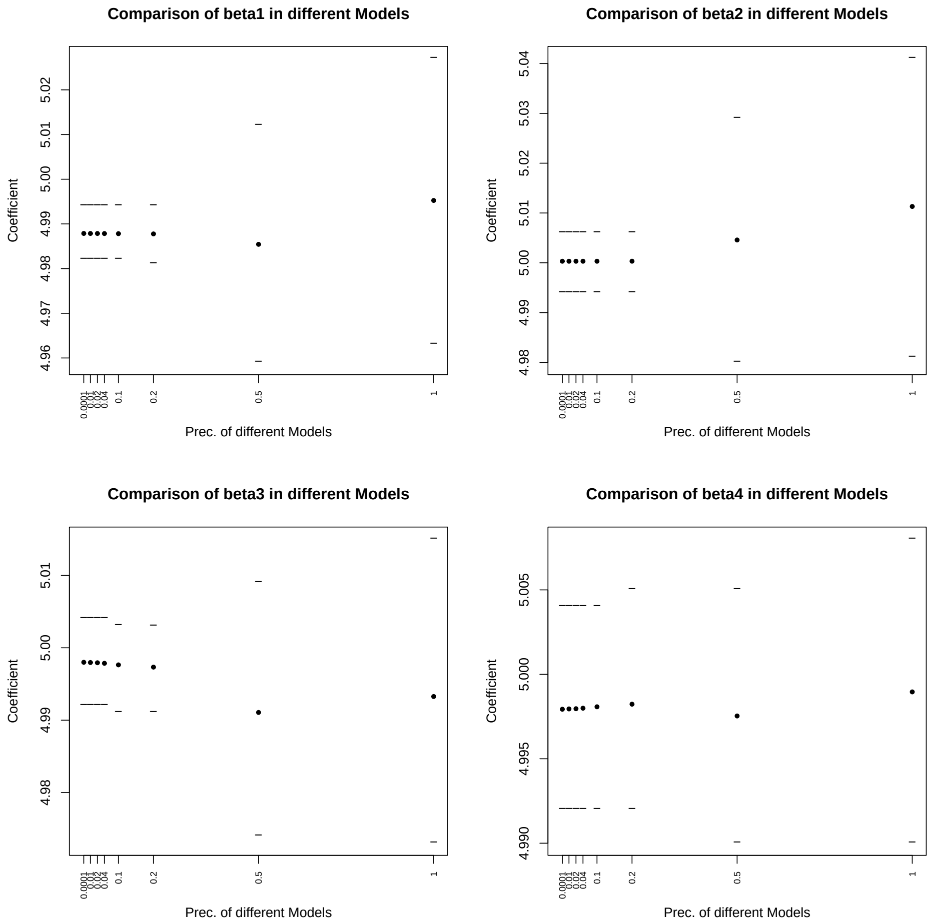

Figure 1: Means and 95% credibilty intervals for parameters of latent variables.

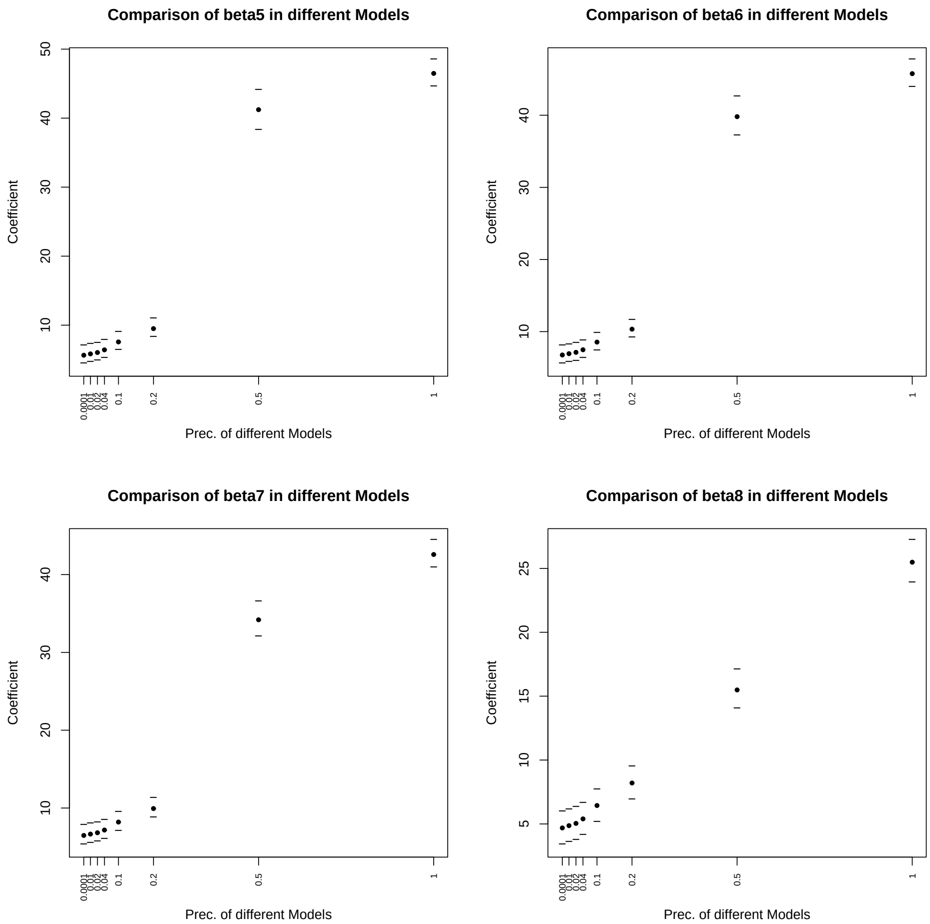

Figure 2: Means and 95% credibility intervals for parameters of dummy variables.

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